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An inverse design problem of estimating optimal shape of cooling passages in turbine blades

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Abstract

An inverse design problem is solved to determine the shape of complex coolant flow passages in internal cooled turbine blades by using the conjugate gradient method (CGM). One of the advantages of using CGM lies in that it can easily handle problems having a huge number of unknown parameters and it converges very fast. The boundary element method (BEM) is used to calculate the direct, sensitivity and adjoint problems due to its characteristics of easily-handling the problem considered here.

Results obtained by using the CGM to solve the inverse problems are verified based on the numerical experiments in the analysis model. One concludes that the CGM is applied successfully in estimating the arbitrary shape of cavities and the rate of convergence is also very fast even when the number of unknown parameters is large. Moreover, the design model of the inverse problem is also performed to estimate the optimal shape of cooling passages in accordance with the desired blade surface temperature distributions.

1. Introduction

The technique in designing the shapes and locations of internal cooling flow passages of turbine blades is very important to turbomachinery industy. Due to its inherent complexity, the design of these cooling passages is usually accomplished using various approximate and empirical techniques.

During the past few years, the pioneers of cooling passage design problems, Dulikravich and his co-workers, have successfully developed inverse design algorithms for estimating the proper sizes, shapes and locations of coolant passages for internally cooled turbine blades and have published a series of interesting papers.

For instance, Kennon and Dulikravich [1,2] used a panel method and Davidon–Fletcher–Powell a method to estimate the shape of internal cooling passages. In that paper, the desired temperature distributions are specified on the inner surfaces (i.e. the surface of cooling passages), where the temperatures are definitely lower than the outer surface (i.e. the surface of the turbine blade). From the designer's viewpoint, the critical regions where high temperature may occur must be on the outer surface. For this reason the desired temperature distributions should be specified on the outer surface. The authors of [1,2] also noticed this reality requirement, and in latter publications the desired quantities are then specified on the outer surfaces.

Dulikravich and Kosovic [3] used the technique of size comparison to minimize the number of cooling

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Nomenclature

B	Biot number
G, H	geometry dependent matrix
$\bar{h}_i$	convective heat transfer coefficient
J	functional defined by Eq. (5)
J'	gradient of functional defined by Eq. (14)
$\bar{K}$	thermal conductivity
$\bar{L}$	reference length
q	heat flux density
$T(x, y)$	estimated dimensionless temperature
$Y(x, y)$	desired dimensionless temperature.

Greek symbols

β	search step size
γ	conjugate coefficient
$\Gamma_i(x, y)$	boundaries of the computational domain
$\delta(\bullet)$	Dirac delta function
$\Delta T(x, y)$	sensitivity function defined by Eq. (7)
ε	convergence criteria
$\lambda(x, y)$	Lagrance multiplier defined by Eq. (12)
ω	random number
Ω	computational domain.

Superscripts

n	iteration index
$\hat{}$	estimated values
$*$	fundamental solution
$\bar{}$	dimensional quantities.

holes. Dulikravich and Martin [4] used superelliptic functions to define the cooling holes and performed inverse estimations. In Refs [3,4] the desired quantities specified on the outer surface are now surface heat fluxes. This is somehow unrealistic since information of temperatures is always much easier to obtain and specify than the heat fluxes on the outer surface.

The boundary conditions used in [1–4] for inner and outer surfaces are the Dirichlet condition, and over-specified (i.e. Dirichlet and Neumann) conditions, respectively. However, for the case of turbine blades considered here, the inner, as well as the outer boundary conditions, should be Robin condition and overspecified (i.e. Dirichlet and Robin) conditions, respectively, to match the real situations since both inner and outer surfaces contain a flow of fluids.

Moreover, the optimization algorithm used in [1–4] is classified as the 'parameter estimation' [5] in inverse problems and did not show explicitly in the paper. As the number of unknown parameters are increased, the rate of convergence becomes very slow since one has to perturb the unknown parameters one at a time to calculate the gradients. This fact can be shown from Ref. [4] where they reported that for the cases of coated hollow disk and airfoil, 2790 and 12,028 s of CPU time is needed, respectively, on an IBM 3090 computer, for only eighteen unknown parameters.

The literature review above indicates that the techniques for [1–4] belong to the parameter estimation [5], therefore, when the number of unknown parameters increased tremendously (such as the case when the number of holes are large), the algorithm may converge very slowly or even may not work. This phenomenon has been discussed by Huang and Chao [6] in an inverse geometry problem. They concluded that the techniques of function estimation would be much better than parameter estimation, since the number of unknown parameters are unlimited when using the technique of function estimation.

Based on the above study, the present work is to develop an inverse design algorithm with the boundary element method (BEM) to estimate the optimal shapes of cooling holes by using the following improvements; (i) the technique of function estimation is used (i.e. the search direction can be obtained by solving only the adjoint problem), (ii) Robin boundary conditions on both inner and outer surfaces are used, and (iii) the desired temperature distributions are specified on the outer surface.

The present paper is actually an extension of the works by Huang and Chao [6] and Huang et al. [7]. In Ref. [6] the search directions are restricted in the y-direction, i.e. the unknown parameters are y-coordinates only. In Ref. [7], we do not confine the search directions, i.e. the unknown parameters become x- and y-coordinates, but only the steepest descent method (SDM) is discussed.

In this paper, the conjugate gradient method (CGM) for the numerical solution to inverse design problems in estimating the optimal locations and shapes of the internal cooling passages for turbine blades based on the desired outer surface temperature distribution are considered.

The use of the boundary element method is suggested by the basic nature of the inverse problem (to search an unknown domain, thus an unknown surface), because domain discretization is avoided. More specifically, the advantages gained by a BEM-based algorithm, is the ability to readily accommodate the changes in the unknown boundary shape as it evolves from its initial, to its final shape, and the ability to handle the problem of multiple internal boundaries.

The present work addresses the developments of the CGM, for estimating unknown shapes of cooling

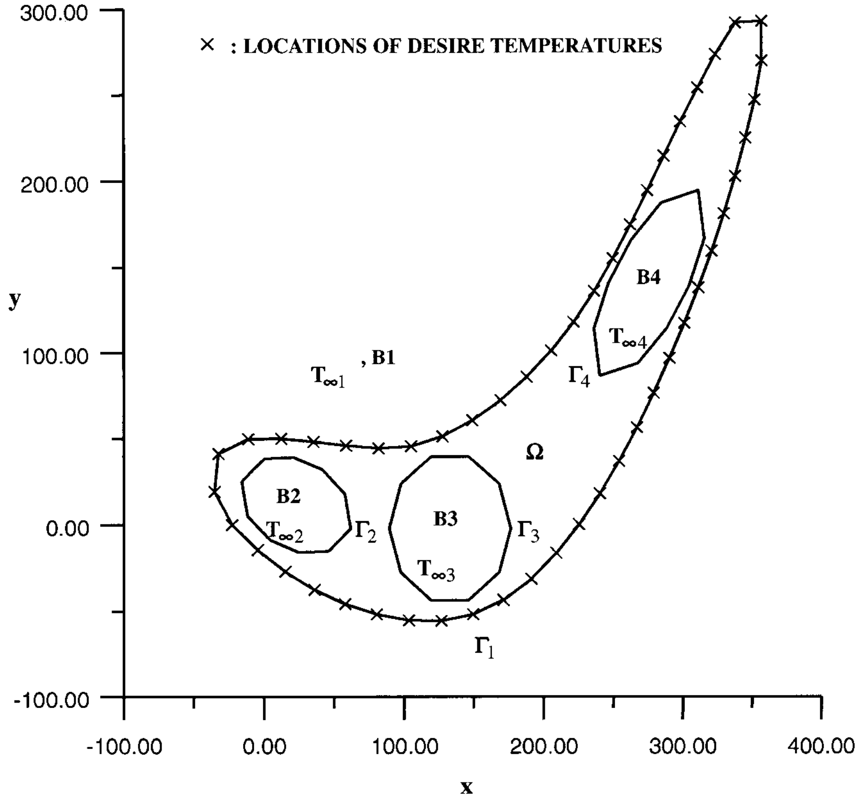

Fig. 1. The system under consideration.

passages. The CGM is derived from the perturbation principle, and transforms the inverse problem to the solution of three problems, namely, the direct problem, the sensitivity problem and the adjoint problem. This method will be discussed in detail in the text.

2. The direct problem

To illustrate the methodology for developing expressions for use in determining the optimal shapes and locations of internal cooling passages for turbine blades in a homogeneous medium with thermal conductivity $\bar{K}$, we consider the following two-dimensional, steady-state, inverse design problem. For a blade with domain Ω, the boundary conditions along outer boundary Γ_1, and inner boundary Γ_i, $i = 2$–I (i.e. there are $(I-1)$ passages inside the turbine blade), are all subjected to the Robin-type boundary conditions, with convective heat transfer coefficients $\bar{h}_i$ and ambient fluid temperatures $\bar{T}_{\infty i}$, $i = 1$–I.

Fig. 1 shows the geometry and the coordinates for the two-dimensional physical problem considered here. The mathematical formulation of this steady-state heat conduction problem in dimensionless form is given by:

$$\frac{\partial^2 T}{\partial^2 x} + \frac{\partial^2 T}{\partial^2 y} = 0 \quad \text{in } \Omega \tag{1a}$$

$$\frac{\partial T}{\partial n} = B_1(T_{\infty 1} - T) \quad \text{along outer boundary } \Gamma_1 \tag{1b}$$

$$\frac{\partial T}{\partial n} = B_i(T - T_{\infty i}) \quad \text{along unknown inner boundary} \quad \Gamma_i \equiv \Gamma_i(x, y), \quad i = 2\text{–}I \tag{1c}$$

where the following dimensionless quantities are defined

$$T = \frac{\bar{T}}{\bar{T}_R}; \quad x = \frac{\bar{x}}{\bar{L}}; \quad y = \frac{\bar{y}}{\bar{L}}; \quad B_i = \frac{\bar{h}_i \bar{L}}{\bar{K}}, \quad i = 1 \text{ to } I$$

here B_i, $i = 1$ to M, represents the Biot number, $\bar{T}_R$ and $\bar{L}$ are the reference temperature and length, respectively. The above problem is solved by the following BEM algorithm.

For a constant property, steady-state heat conduction problem with a domain Ω and boundary Γ, the boundary integral equation for this problem without generation term can be derived as [8]

$$cT_{\mathrm{m}} + \int_{\Gamma} Tq^* \, \mathrm{d}\Gamma = \int_{\Gamma} qT^* \tag{2}$$

where m = point on Γ or in Ω, T = temperature, $q = (\partial T/\partial n)$ = heat flux density, $c = 1$, if m is in Ω, $c < 1$ if m is on Γ ($c = 0.5$ if Γ is smooth at m), T^* = stationary fundamental solution, q^* = normal derivative of T^*.

Thus,

$$T^* = \frac{1}{2\pi} \ln \left(\frac{1}{r}\right) \quad \text{in two dimensions}$$

where r = distance from m to a point of Γ.

Generally speaking, the discretization of Γ into k boundary elements allows substitution into the boundary integral Eq. (2) expressed for each boundary element of the algebraic linear system [8]

$$\mathbf{CT} + \mathbf{HT} = \mathbf{Gq} \tag{3}$$

where $\mathbf{T}$ = vector of temperature boundary elements, $\mathbf{q}$ = vector of boundary heat flux densities, $\mathbf{H}$, $\mathbf{G}$ = geometry dependent matrices, $\mathbf{C}$ = diagonal matrix.

Once all unknowns are passed to the left-hand-side and the knowns are gathered on the right-hand-side, one can write

$$\mathbf{AX} = \mathbf{B} \tag{4}$$

where $\mathbf{X}$ is the vector of unknown Ts and qs on the boundary. $\mathbf{B}$ is found by multiplying the corresponding columns by the known values of Ts or qs.

The computer program for the above problem is modified, based on the textbook by Brebbia and Dominguez [8], and linear boundary elements are adopted for all the examples illustrated here.

The direct problem considered here is concerned with the determination of the medium temperature when the locations and shapes of inner cooling passages $\Gamma_i(x, y)$, $i = 2$–I, and the conditions at all boundaries are known.

3. The inverse design problem

For the inverse problem, the locations and shapes of inner cooling passages $\Gamma_i(x, y)$, $i = 2$–I, are regarded as being unknown, but everything else in Eq. (1) is known. In addition, desired temperature distributions on the outer blade surface Γ_1 are considered available. This implies that the dual boundary conditions (i.e. Dirichlet and Robin) are specified on the surface Γ_1.

Referring to Fig. 1, we assumed that the number of locations for the specified desired temperatures on Γ_1 is M (marked by the symbol '$\times$' in Fig. 1). The purpose of the present study is to use those M temperature data points to estimate the optimal locations and shapes of inner cooling passages $\Gamma_i(x, y)$, $i = 2$–I in the inverse design calculations.

Let the desired temperature distributions taken on Γ_1 be denoted by $Y(x_m, y_m) \equiv Y_m$, $m = 1$–M, where M represents the number of desired temperatures. Then the inverse problem can be stated as follows: by utilizing the above mentioned desired temperature data Y_m, estimate the unknown locations and shapes of the internal cooling passages $\Gamma_i(x, y)$, $i = 2$–I.

The solution of the present inverse design problem is obtained in such a way that the following functional is minimized:

$$J[\hat{\Gamma}_i(x, y)] = \sum_{m=1}^{M} [T_m - Y_m]^2 \tag{5}$$

where T_m are the estimated or computed desired temperatures on the outer blade surface Γ_1. These quantities are determined from the solution of the direct problem given previously by using an estimated $\hat{\Gamma}_i(x, y)$ for the exact $\Gamma_i(x, y)$. Here the hat '$\hat{\ }$' denotes the estimated quantities.

4. Conjugate gradient method CGM for minimization

The SDM [7] is similar to, but simpler than the CGM [9] since the calculations of the conjugate coefficient and direction of descent are not needed. However, the drawback for SDM is that the rate of convergence is slower than the CGM.

The following iterative process based on the CGM is now used for the estimation of the unknown boundary shapes $\Gamma_i(x, y)$ by minimizing the functional $J[\hat{\Gamma}_i(x, y)]$.

$$\hat{\Gamma}_i^{n+1}(x, y) = \hat{\Gamma}_i^{n}(x, y) - \beta^n P_i^n(x, y) \quad \text{for } i = 2\text{–}I \text{ and } n = 0, 1, 2, \ldots \tag{6a}$$

or more explicitly

$$\hat{x}^{n+1} = \hat{x}^n - \beta^n P_i^n(x, y) \times \cos\phi \tag{6b}$$

$$\hat{y}^{n+1} = \hat{y}^n - \beta^n P_i^n(x, y) \times \sin\phi \tag{6c}$$

and

$$\hat{\Gamma}_i^{n+1}(x, y) = \hat{\Gamma}_i(\hat{x}^{n+1}, \hat{y}^{n+1}), \quad i = 2\text{–}I \tag{6d}$$

where ϕ is the angle between horizontal and normal outward direction of the unknown boundary as shown in Fig. 2. The value of ϕ can be calculated for any

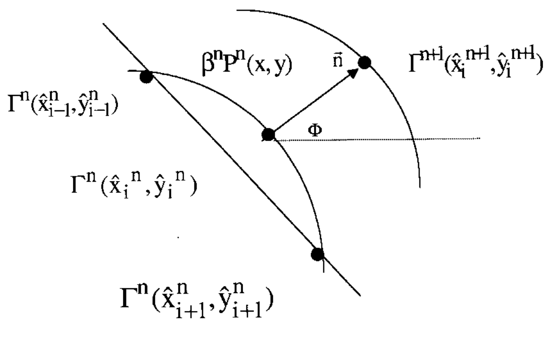

Fig. 2. The graphical analysis of CGM from n to $(n+1)$ iterations.

given configurations. β^n is the search step size in going from iteration n to iteration $n+1$ and $P_i^n(x, y)$ is the direction of descent (i.e. search direction) given by

$$P_i^n(x, y) = J_i^{\prime n}(x, y) + \gamma^n P_i^{n-1}(x, y), \quad i = 2\text{–}I \tag{6e}$$

which is a conjugation of the gradient in the outward normal direction $J_i^{\prime n}(x, y)$ at iteration n and the direction of descent $P_i^{n-1}(x, y)$ at iteration $n-1$. The conjugate coefficient is defined as [9]

$$\gamma_i^n = \frac{\int_{\Gamma_i} (J_i^{\prime n})^2 \, \mathrm{d}\Gamma_i}{\int_{\Gamma_i} (J_i^{\prime n-1})^2 \, \mathrm{d}\Gamma_i} \quad \text{with } \gamma_i^0 = 0, \quad i = 2\text{–}I \tag{6f}$$

We note that when $\gamma_i^n = 0$ for any n, in Eq. (6f), the direction of descent $P^n(x, y)$ becomes the gradient direction, i.e. the SDM is obtained. The convergence of the above iterative procedure in minimizing the functional J is guaranteed in [10].

To perform the iterations according to Eq. (6a), we need to compute the step size β^n and the gradient of the functional $J_i^{\prime n}(x, y)$. In order to develop expressions for the determination of these two quantities, a 'sensitivity problem' and an 'adjoint problem' are constructed as described below.

5. Sensitivity problem and search step size

The sensitivity problem is obtained from the original direct problem defined by Eq. (1) in the following manner: it is assumed that when $\Gamma_i(x, y)$, $i = 2\text{–}I$, undergoes a variation $\Delta\Gamma_i(x, y)$, $T(x, y)$ is perturbed by ΔT. Then replacing in the direct problem $\Gamma_i(x, y)$ by $\Gamma_i(x, y) + \Delta\Gamma_i(x, y)$ and T by $T + \Delta T$, subtracting from the resulting expressions the direct problem and neglecting the second-order terms, the following sensitivity problem for the sensitivity function ΔT are obtained.

$$\frac{\partial^2 \Delta T}{\partial x^2} + \frac{\partial^2 \Delta T}{\partial y^2} = 0 \quad \text{in } \Omega \tag{7a}$$

$$\frac{\partial \Delta T}{\partial n} = -B_1 \Delta T \quad \text{along outer boundary } \Gamma_1 \tag{7b}$$

$$\frac{\partial \Delta T}{\partial n} = B_i \Delta\Gamma_i \frac{\partial T}{\partial n} \quad \text{along unknown inner boundary} \quad \Gamma_i \equiv \Gamma_i(x, y), \quad i = 2\text{–}I \tag{7c}$$

where $\partial T/\partial n$ represents the temperature gradient along the normal direction of Γ_i. The BEM technique is used to solve this sensitivity problem.

The functional $J(\hat{\Gamma}_i^{n+1})$ for iteration $n+1$ is obtained by rewriting Eq. (5) as

$$J[\hat{\Gamma}_i^{n+1}] = \sum_{m=1}^{M} \Big[T_m(\hat{\Gamma}_i^n - \beta^n P_i^n) - Y_m \Big]^2 \tag{8a}$$

where we replaced $\hat{\Gamma}_i^{n+1}$ by the expression given by Eq. (6a). If temperature $T_m(\hat{\Gamma}_i^n - \beta^n P_i^n)$ is linearized by a Taylor expansion, Eq. (8a) takes the form

$$J(\hat{\Gamma}_i^{n+1}) = \sum_{m=1}^{M} \Big[T_m(\hat{\Gamma}_i^n) - \beta^n \Delta T_m(P_i^n) - Y_m \Big]^2 \tag{8b}$$

where $T_m(\hat{\Gamma}_i^n)$ is the solution of the direct problem on Γ_i by using estimate $\hat{\Gamma}_i^n(x, y)$ for exact $\Gamma_i(x, y)$. The sensitivity functions $\Delta T_m(P_i^n)$ are taken as the solutions of problem (7) on $\Gamma_i(x, y)$ by letting $\Delta\Gamma_i = P_i^n$. The search step size β^n is determined by minimizing the functional given by Eq. (8b) with respect to β^n. The following expression results:

$$\beta^n = \frac{\sum_{m=1}^{M} (T_m - Y_m) \Delta T_m}{\sum_{m=1}^{M} (\Delta T_m)^2} \tag{9}$$

6. Adjoint problem and gradient equations

To obtain the adjoint problem, Eq. (1a) is multiplied by the Lagrange multiplier (or adjoint function) $\lambda(x, y)$ and the resulting expression is integrated over the corresponding space domain. Then the result is added to the right-hand-side of Eq. (5) to yield the following expression for the functional $J[\hat{\Gamma}_i^n(x, y)]$:

$$J[\hat{\Gamma}_i(x,y)] = \sum_{m=1}^{M}[T_m - Y_m]^2$$

$$+\int_{\Omega}\lambda\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)\mathrm{d}\Omega \tag{10}$$

The variation ΔJ is obtained be perturbing Γ_i by $\Delta\Gamma_i$ and T by ΔT in Eq. (1), subtracting from the resulting expression the original Eq. (1) and neglecting the second-order terms. We thus find

$$\Delta J = \int_{\Gamma_1} 2(T-Y)\Delta T\delta(x-x_m)\delta(y-y_m)\,\mathrm{d}\Gamma_1$$

$$+\int_{\Omega}\lambda\left[\frac{\partial^2 \Delta T}{\partial x^2}+\frac{\partial^2 \Delta T}{\partial y^2}\right]\mathrm{d}\Omega \tag{11}$$

where $\delta(\cdot)$ is the Dirac delta function and (x_m, y_m), $m=1$ to M, refer to the surface points on Γ_1 where the desired temperatures are specified. In Eq. (11), the domain integral term is reformulated based on Green's second identity; the boundary conditions of the sensitivity problem given by Eqs. (7b) and (7c) are utilized and then ΔJ is allowed to go to zero. The vanishing of the integrands containing ΔT leads to the following adjoint problem for the determination of $\lambda(x, y)$:

$$\frac{\partial^2 \lambda}{\partial x^2}+\frac{\partial^2 \lambda}{\partial y^2} = 0 \quad \text{in } \Omega \tag{12a}$$

$$\frac{\partial \lambda}{\partial n} + B_1\lambda = 2(T-Y)\,\delta(X-X_m)\delta(y-y_m) \quad \text{along outer boundary } \Gamma_1 \tag{12b}$$

$$\frac{\partial \lambda}{\partial n} = 0 \quad \text{along unknown inner boundary } \Gamma_i \equiv \Gamma_i(x,y),\ i=2\text{–}I \tag{12c}$$

The standard techniques of BEM can be used to solve the above adjoint problem. Finally, the following integral term is left

$$\Delta J = \int_{\Gamma_i}\left[B_i\lambda\frac{\partial T}{\partial n}\right]\Delta\Gamma_i(x,y)\,\mathrm{d}\Gamma_i \tag{13a}$$

From definition [9], the functional increment can be presented as

$$\Delta J = \int_{\Gamma_i} J'_i(x,y)\Delta\Gamma_i(x,y)\,\mathrm{d}\Gamma_i \tag{13b}$$

A comparison of Eqs. (13a) and (13b) leads to the following expression for the gradient $J'_i(x, y)$ of the functional $J[\hat{\Gamma}_i(x, y)]$:

$$J'_i(x,y) = B_i\lambda\frac{\partial T}{\partial n}\bigg|_{\Gamma_i}, \quad i=2\text{–}I \tag{14}$$

The calculation of gradient equations is the most important part of CGM since it plays a significant role of the inverse calculation.

7. Computational procedure

The computational procedure for the solution of this inverse problem using the conjugate gradient method may be summarized as follows:

Suppose $\hat{\Gamma}_i^n(x, y)$ is available at iteration n.

1. Solve the direct problem given by Eq. (1) for $T(x, y)$.
2. Examine the stopping criterion with a specified ε. Continue if not satisfied.
3. Solve the adjoint problem given by Eq. (12) for $\lambda(x, y)$.
4. Compute the gradient of the functional $J_i'^n$ from Eq. (14).
5. Compute the conjugate efficient γ_i^n and direction of descent P_i^n from Eqs. (6f) and (6e), respectively.
6. Set $\Delta\Gamma_i(x, y) = P_i^n(x, y)$, and solve the sensitivity problem given by Eq. (7) for $\Delta T(x, y)$.
7. Compute the search step size β^n from Eq. (9).
8. Compute the new estimation for $\hat{\Gamma}_i^{n+1}(x, y)$ from Eq. (6 d) and functional J from Eq. (5). If the value of the objective function J is less than the specified stopping criteria ε, stop the iterations, otherwise, return to Step 1.

8. Results and discussions

The objective of this article is to show the validity of the present approaches in estimating $\Gamma_i(x, y)$, $i = 2$–I, where $(I-1)$ is the number of cooling passages, with no prior information on the functional form of the unknown cavities, which is the so-called function estimation. To illustrate the ability of the present inverse algorithm in estimating the optimal shape and location of the cooling passages for turbine blade from the knowledge of the specified desired temperatures, we consider following two specific models, i.e. the analysis model and the design model.

In the analysis model, the exact shapes of the cooling passages are given, then the temperatures on the outer surface can be calculated and specified as the desired temperatures. By using these desired temperatures and arbitrary initial guesses of the holes, one is asked to reconstruct the exact shapes of cooling passages by the conjugate gradient method in the inverse

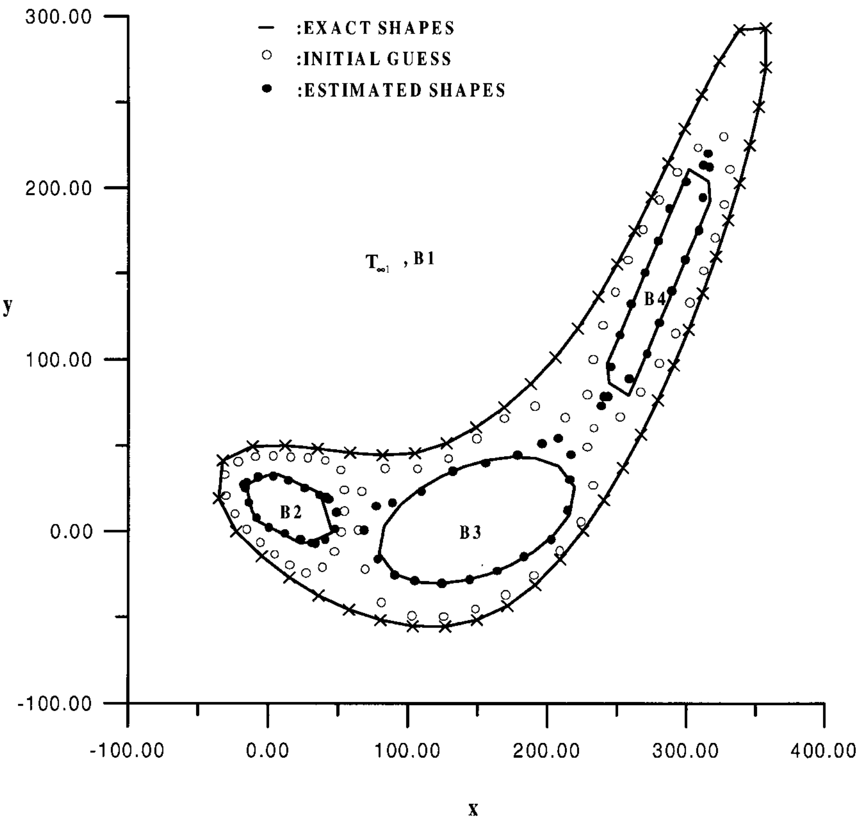

Fig. 3. The exact and estimated cooling passages in the analysis model.

design problem. Under this consideration the objective function may decrease to a very small number since there exists an exact solution.

In the design model, two design problems are considered (i) the desired temperatures on the blade surface are modified from the existing blade surface temperatures, (ii) the desired temperatures on the blade surface are the same as the exiting blades surface temperatures but the Biot number of the cooling passages are changed. The objective now is to estimate the shapes and locations of the cooling holes to minimize the objective function. Under this situation the objective function may not decrease to a small number since no exact solution exists, however, the optimal solution can still be obtained.

We now present below, two models in determining $\Gamma_i(x, y)$ by the inverse analysis.

8.1. Analysis model

The objective of the analysis model is to show the validity of the present inverse design algorithm in estimating the shape of cooling holes. For this reason the following numerical simulation is performed.

The exact configuration of the turbine blade with three internal cooling passages (i.e. $I = 4$) is shown in Fig. 3. The ambient fluid temperature and Biot number for the blade surface on Γ_1 are taken as 1000 and 2.0, respectively. The ambient fluid temperatures and Biot number for cooling holes on Γ_2, Γ_3 and Γ_4 are assumed the same and taken as 10 and 0.1, respectively. The reference temperature, $\bar{T}_R$, and reference length, $\bar{L}$, are both taken as unity for convenience. The number of linear elements used for Γ_1, Γ_2, Γ_3 and Γ_4 are 50, 20, 20 and 20, respectively, which implies that the unknown parameters of x- and y-coordinates are 120 in the present case. The number of unknown parameters is now larger than was calculated in Ref. [4].

The inverse analysis is then performed based on 50 desired temperature data on Γ_1 (referring to Fig. 3 where the symbol ' $\times$ ' denotes the location for the desired temperatures) i.e. $Y(x_m, y_m) \equiv Y_m$, $m = 1$ to 50. Those desired temperatures Y_m are obtained by using the given boundary conditions and the exact shape of cooling passages (as shown in Fig. 3). Once

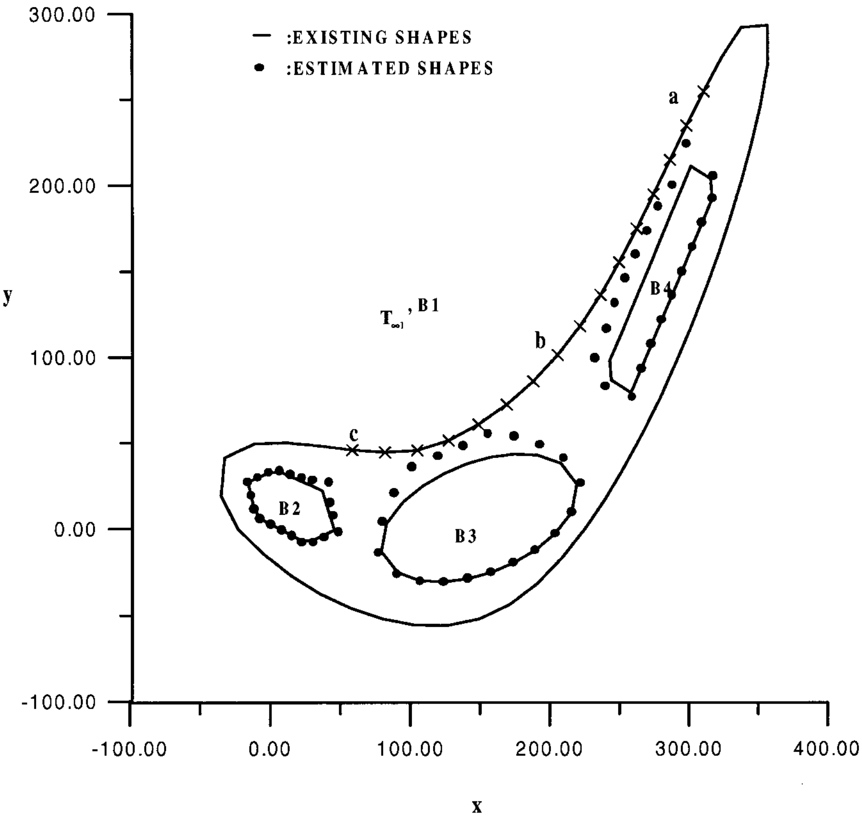

Fig. 4. The estimated cooling passages for lower desired surface temperature along $\overline{abc}$ curve in the design model.

the desired temperatures Y_m are obtained, our objective is to reconstruct the exact shape of cooling passages by using those Y_m and any arbitrary initial guesses for the cooling holes (referring to Fig. 3 where the symbol '○' denotes the initial guess for the shape of the cooling passages). Here, the initial guesses for the cooling holes are chosen as close to the blade surface as possible.

At the first iteration, the value of the objective functional is calculated as $J = 1277$ and the average absolute error for the estimated temperature is calculated as ERR = 4.6. After only nine iterations, the value of the objective functional is decreased to $J = 8$ and ERR = 0.2.

Here the definition for the average absolute error (ERR) is given as

$$\mathrm{ERR} = \left[\sum_{m=1}^{M} | T_m - Y_m | \right] \div (M) \tag{15}$$

where M represents the total number of desired temperature data, while T_m and Y_m denote the estimated and desired values of blade surface temperatures.

The estimated shape and location for the cooling passages is shown in Fig. 3 with symbol '●' and they are indeed, in good agreement with the exact shapes except for a few points at the center of the blade. The reason for this is because the distance from the center points to the blade surface is large and therefore, the change of shape at the center region can hardly effect the temperature on the blade's outer surface. A similar phenomenon has been observed by [11] where they called this a 'corner effect'.

The computer time needed for the above calculation on a 586–266 MHz (Pentium II-266 CPU) PC, is about 12 s. This implied that the present algorithm only needs a very short computer time to perform the inverse calculations in estimating 120 unknown parameters simultaneously.

The above numerical experiment shows the validity of CGM in inverse calculation. Next our task is to test the ability of the present algorithm in performing the design process.

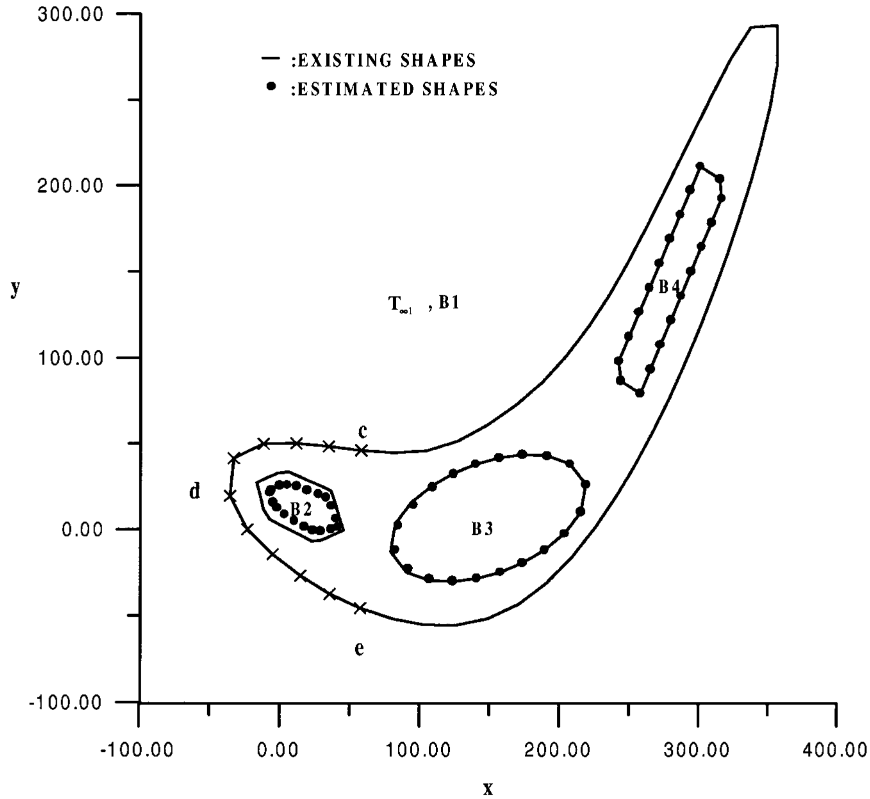

Fig. 5. The estimated cooling passages for higher desired surface temperature along $\overline{cde}$ curve in the design model.

8.2. Design model

The existing turbine blade with internal cooling holes is taken as the exact shape shown in Fig. 3 as were used in the analysis model. In the design model, the following two design problems will be discussed.

8.2.1. Problem 1

In Problem 1, the shape of the cooling passages for the existing blade is the same as the exact passages in the analysis model. The working conditions for Problem 1 are taken as $T_{\infty 1} = 1000$, $T_{\infty 2} = T_{\infty 3} = T_{\infty 4} = 10$ and $B_1 = 2.0$, $B_2 = B_3 = B_4 = 0.5$. The desired temperatures on the blade surface are modified from the existing blade surface temperatures, then by using those desired temperatures one is asked to estimate the optimal shape of the cooling passages that minimized the objective function. Under this consideration, no exact shape of cooling holes exists, but the best shapes can be obtained.

Three test cases will be demonstrated in Problem 1, they are (a) lower surface temperatures are required, (b) higher surface temperatures are required and (c) half of the surface required higher surface temperatures while the other half required lower surface temperatures.

In the first test case the temperatures on the portion of blade surface Γ_1 along $\overline{abc}$ curve (marked by the symbol ' × ' in Fig. 4) are required to be decreased by 10 from the existing surface temperatures while the rest of the surface temperatures are kept the same. The objective function is set accordingly and the program for the inverse design analysis is performed. Initially the value of objective function $J = 1600$ and $\mathrm{ERR} = 10$. After seventeen iterations (CPU time is about 30 s) we have $J = 588$ and $\mathrm{ERR} = 5.7$. The estimated optimal profile for the internal passages is shown in Fig. 4 with symbol '●'.

From Fig. 4 we learned that the cooling holes are enlarged near the boundaries $\overline{abc}$ where the temperatures are required to be decreased while kept almost the same for the rest of the holes. This implies that we need to move the cooling passages closer to the boundary $\overline{abc}$ to lower the surface temperatures along $\overline{abc}$, and this is a reasonable implication. The objective function J cannot decrease to a small number as in the analysis model, this is because in a realistic situation

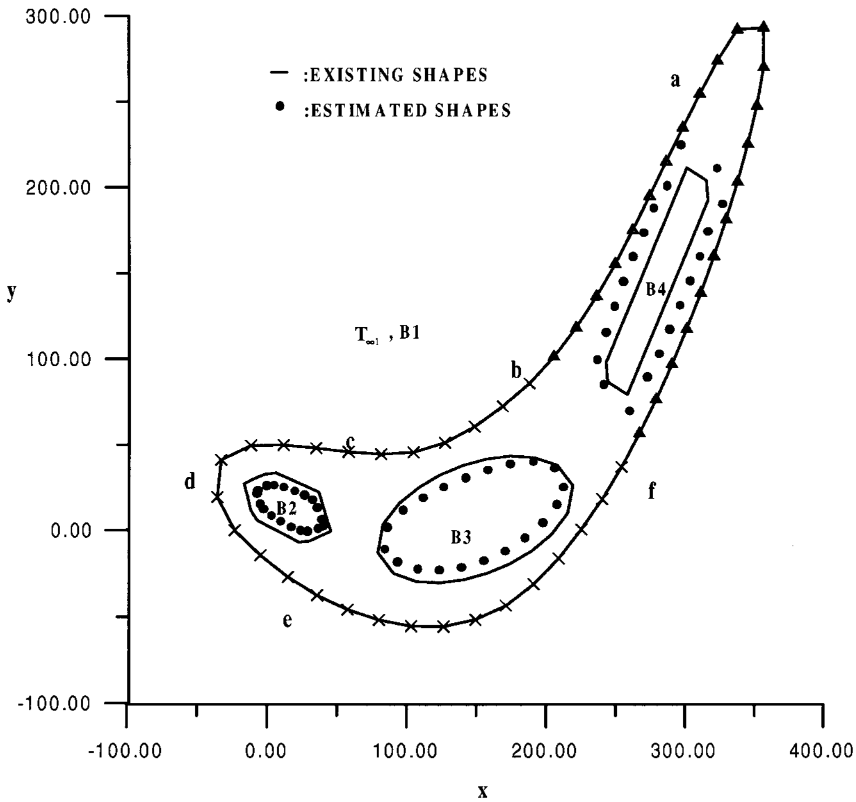

Fig. 6. The estimated cooling passages for higher desired temperature along $\overline{bdf}$ and lower desired temperature along $\overline{fab}$ in the design model.

we cannot find the cooling passages that exactly match the desired temperature distributions.

In the second test case the temperatures on the portion of the blade surface Γ_1 along $\overline{cde}$ curve (marked by the symbol ' × ' in Fig. 5) are required to be increased by eight from the existing surface temperatures while the rest of the surface temperatures are kept the same. Initially the value of objective function $J = 704$ and ERR = 8. After four iterations (CPU time is about 9 s) we have $J = 360$ and ERR = 5.8. The estimated optimal profile for the internal passages is shown in Fig. 5 with symbol '●'.

It can be seen from Fig. 5 that the cooling holes are shrunk near the boundaries $\overline{cde}$ where the temperatures are required to be increased while kept almost the same for the rest of the holes. This means we need to move the cooling passages far away from the boundary $\overline{cde}$ to increase the surface temperatures along $\overline{cde}$.

In the third test case the temperatures along $\overline{bdf}$ curve (marked by the symbol ' × ' in Fig. 6) are required to be increased by eight and the temperatures along $\overline{fab}$ curve (marked by the symbol 'Δ' in Fig. 6) are required to be decreased by 10 from the existing surface temperatures. Initially the value of objective function $J = 4028$ and ERR = 8.92. After 10 iterations (CPU time is about 20 s) we have $J = 1863$ and ERR = 5.4. The estimated optimal profile for the internal passages is shown in Fig. 6 with symbol '●'.

It is obvious from Fig. 6 that the cooling holes are now shrunk around the boundaries $\overline{bdf}$ where the temperatures are required to increase, while they are enlarged around the boundaries $\overline{fab}$ where the temperatures are required to decrease.

8.2.2. Problem 2

In Problem 2, if the Biot number for the internal fluid is subjected to be changed, i.e. different from the existing working condition, but the existing surface temperature distribution is asked to be kept, i.e. the desired temperatures on the blade surface are the same as the existing blade surface temperatures. The location of desired temperatures is marked by the symbol ' × ' in Figs. 7 and 8. Under this consideration one is asked to estimate the optimal shape of the cooling passages that minimized the objective function.

Two test cases will be discussed in Problem 2, they

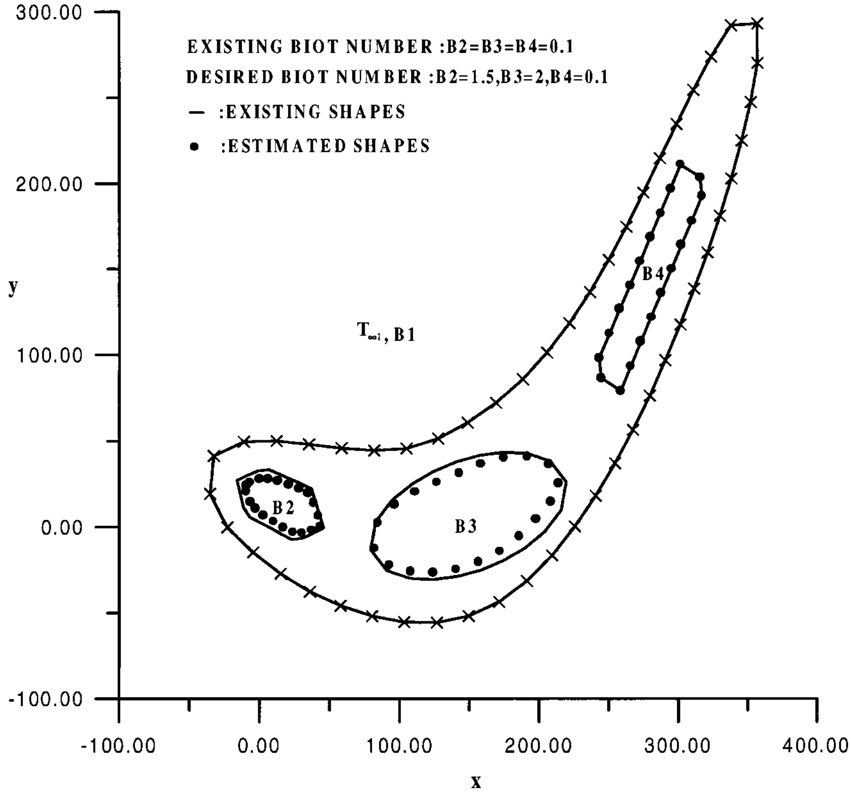

Fig. 7. The estimated cooling passages for higher Biot number on Γ_2 and Γ_3 holes in the design model.

are (a) the Biot numbers for the inner holes are increased and (b) the Biot numbers for the inner holes are decreased.

In the first case the existing working conditions are taken as $T_{\infty 1}=1000$, $T_{\infty 2}=T_{\infty 3}=T_{\infty 4}=10$, $B_1=2.0$, $B_2=B_3=B_4=0.1$. Now we are requested that the Biot number of the internal cooling passages are changed to $B_2=1.5$, $B_3=2$ and $B_4=0.1$. The inverse design program is then performed. Initially the value of objective function J = 584 and ERR = 2.3. After 20 iterations (CPU time is about 30 s) we have J = 130 and ERR = 1.1. The estimated optimal profile for the internal passages is shown in Fig. 7 with symbol '●'.

It can be seen from Fig. 7 that the cooling holes are shrunk for the first and second passages and kept almost the same for the third passage. This is because when the Biot number is increased the temperature of the inner surface should be decreased. In order to keep the same temperature distribution on the blade surface, the cooling passages must be shrunk.

In the second case, the existing working conditions are taken as $T_{\infty 1}=1000$, $T_{\infty 2}=T_{\infty 3}=T_{\infty 4}=10$, $B_1=2.0$, $B_2=B_3=B_4=1$. Now the Biot number of the internal cooling passages are asked to change to $B_2=0.3$, $B_3=1$ and $B_4=0.2$. The inverse design program is then performed again. Initially the value of objective function j = 127 and ERR = 1.1. After 20 iterations (CPU time is about 30 s) we have J = 1.3 and ERR = 0.01. The estimated optimal profile for the internal passages is shown in Fig. 8 with symbol '●'.

It is clear from Fig. 8 that the cooling holes are enlarged for the first and third passages and kept almost the same for the second passage. This is because when the Biot number is decreased, the temperature of the inner surface should be increased. In order to keep the same temperature distribution on the blade surface, the cooling passages must be enlarged. Finally the information for all the above results are summarized in Table 1.

From the above discussions we conclude that the advantages of using the conjugate gradient method are, that (i) it does not require a very accurate initial guess of the unknown shapes, and (ii) the rate of convergence is very fast since it belongs to the technique of functions estimations.

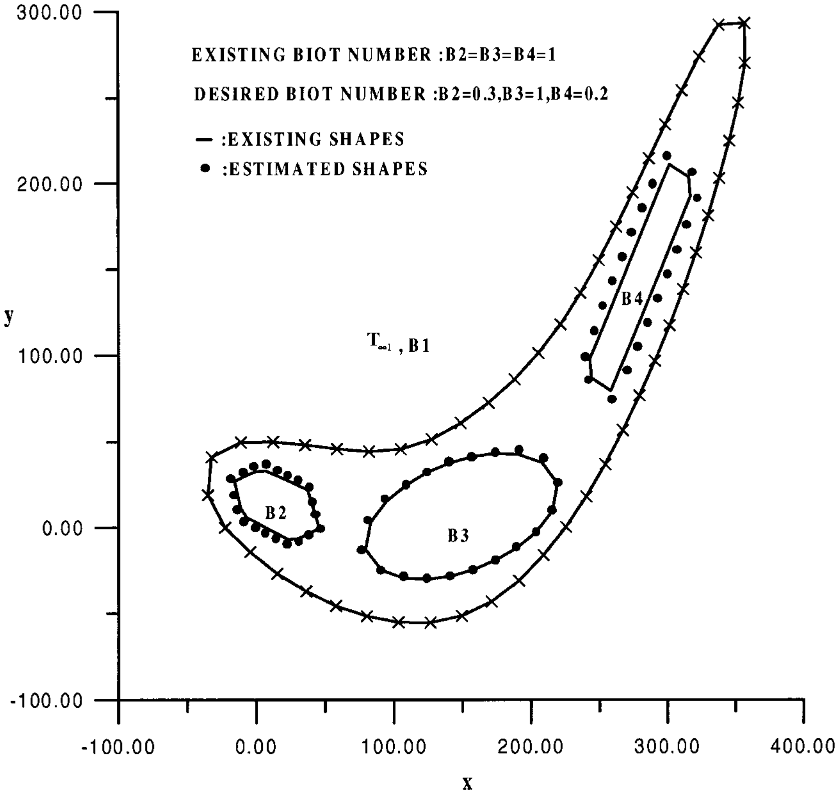

Fig. 8. The estimated cooling passages for lower Biot number of Γ_2 and Γ_4 holes in the design model.

9. Conclusions

The CGM together with the BEM were successfully applied for the solution of the inverse design problem to estimate the optimal shape of the internal cooling passages in turbine blades. Several test cases involving different design considerations were examined.

The results shown that the use of CGM in estimating the unknown shape of the cooling holes has the following advantages. (i) It does not require an accurate initial guess of the unknown quantities (as was shown in the analysis model), (ii) it does not need any assumptions for the functional form of the cooling shape, such as assumed in [4], and (iii) it needs very short CPU time on the Pentium II-266 MHz PC to complete the calculations.

Acknowledgement

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Table 1
The information for all the numerical experiments

	Initial value of J	Initial value of ERR	Final value of J	Final value of ERR	Number of iteration	CPU time, s
Fig. 3	1277	4.6	8	0.2	9	12
Fig. 4	1600	10	588	5.7	17	30
Fig. 5	704	8	360	5.8	4	9
Fig. 6	4028	8.92	1863	5.4	10	20
Fig. 7	584	2.3	130	1.1	20	30
Fig. 8	127	1.1	1.3	0.01	20	30

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